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Spherically symmetric solution of gauge supersymmetry equations

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Abstract. The Einstein–Maxwell field equations of gauge supersymmetry are solved. We find that certain auxiliary gauge fields contribute as much to the energy–momentum tensor as the electromagnetic field itself and give rise to negative energy densities, unless they grow a superheavy mass by spontaneous symmetry breaking.

1. Introduction

Supersymmetry (Volkov and Soroka 1973, Wess and Zumino 1974, Zumino 1974, Salam and Strathdee 1974a, b, 1975) which considers transformations mixing fermions and bosons, opens up the possibility that particles other than vector mesons can be gauge particles in models unifying two or more fundamental interactions. Salam and Strathdee (1974a, b, 1975) introduced the use of linear transformations in an 8-dimensional superspace, $z^A = \{x^\mu, \theta^i\}$ with x^μ being Bose space–time coordinates and θ^i anti-commuting Fermi coordinates, to describe the usual supersymmetry transformation. Arnowitt and Nath (Arnowitt *et al* 1975, Arnowitt and Nath 1975, 1976, Nath and Arnowitt 1975, 1976, Nath 1976) extended supersymmetry to be a local gauge invariance by considering arbitrary coordinate transformations in superspace which leave the line element $ds^2 = dz^A g_{AB}\, dz^B$ invariant. The Einstein-like field equations $R_{AB} = 0$ with spontaneous symmetry breaking appear capable of describing the combined gravity–scalar meson field, gravity–Maxwell field, Maxwell–Dirac fields, matter–Yang–Mills fields, and perhaps even fairly realistic strong interactions using colour gluons depending on the choice of Fermi coordinates θ^i (gauge group). The theory is very attractive in that all the fields in the theory are gauge fields so that the theory is completely self-sourced.

In the present paper we will investigate the Einstein–Maxwell case and find solutions of the gauge supersymmetry equations corresponding to static, spherical symmetry. The Fermi coordinates in this case are doublets of Majorana spinors θ^{iq} with $q = 1, 2$. We will find that the complete energy–momentum tensor which acts as a source of Einstein's field equations has unphysical negative energy densities when all of the auxiliary gauge fields are included, unless further spontaneous symmetry breaking occurs.

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2. Gauge supersymmetry field equations

The field equations of Arnowitt and Nath (1975) can be written out for the Einstein–Maxwell case as

$$G_{\mu\nu} = -8(p_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}p^{\lambda}{}_{\lambda}) \tag{1}$$

$$K_{\mu\nu}(p_{\lambda\rho}) \equiv p_{\mu}{}^{\lambda}{}_{;\lambda\nu} + p_{\nu}{}^{\lambda}{}_{;\lambda\mu} - p_{\mu\nu;}{}^{\lambda}{}_{\lambda} - p_{\lambda}{}^{\lambda}{}_{;\mu\nu} = \tfrac{1}{2}e^2F_{\mu}{}^{\lambda}F_{\nu\lambda} - \tfrac{1}{2}f_{;\mu\nu} \tag{2}$$

$$-\Box^2 f = \tfrac{1}{2}e^2F^{\lambda\rho}F_{\lambda\rho} \tag{3}$$

and

$$F_{\mu}{}^{\lambda}{}_{;\lambda} = 0. \tag{4}$$

Here $G_{\mu\nu}$ is the Einstein tensor and $F^{\lambda\rho}$ the electromagnetic field tensor. $p_{\mu\nu}$ and $f(x)$ are gauge fields arising in the Bose–Bose sector and Fermi–Fermi sector of the expansion of the metric g_{AB} respectively. In (1)–(4), $g_{\mu\nu}$ is the usual Einstein metric tensor and a semicolon denotes covariant differentiation with respect to this metric. Arnowitt and Nath (1976) have shown that spontaneous symmetry breaking really occurs for this system so that $p_{\mu\nu}$ grows a mass, and a term $-m_G^2p_{\mu\nu}$ appears on the left-hand side of (2). After spontaneous symmetry breaking (1) can be written as

$$G_{\mu\nu} = 8\pi G[T_{\mu\nu(\mathrm{EM})} - 2(K_{\mu\nu}(q) - \tfrac{1}{2}g_{\mu\nu}K^{\lambda}{}_{\lambda}(q)) - (h_{;\mu\nu} - g_{\mu\nu}\Box^2 h)] \tag{5}$$

where $G \equiv e^2/2\pi m_G^2$ is the gravitational constant and new fields $h \equiv f/e^2$ and $q_{\mu\nu} \equiv p_{\mu\nu}/e^2$ have been introduced for convenience. Note that a sign misprint in Arnowitt and Nath (1975) has been corrected in front of the h term in (5). In terms of h and $q_{\mu\nu}$ we have

$$K_{\mu\nu}(q) - m_G^2q_{\mu\nu} = \tfrac{1}{2}F_{\mu}{}^{\lambda}F_{\nu\lambda} - \tfrac{1}{2}h_{;\mu\nu} \tag{6}$$

and

$$\Box^2 h = -\tfrac{1}{2}F^{\lambda\rho}F_{\lambda\rho}. \tag{7}$$

To put (5) in the form given, with the electromagnetic energy–momentum tensor correctly appearing on the right-hand side, (3) was used above.

Gravitation and electromagnetism can only be said to be unified correctly in this formalism, if it can be shown that the $K_{\mu\nu}$ and h terms in (5) make no contribution to observable physics. We first note that the divergences of the $K_{\mu\nu}$ and h terms in (5) vanish identically in the flat-space limit. In curved space these divergences do not vanish (energy is not conserved) but fields of order G^2 arising from higher-order terms in g_{AB} have *already* been neglected on the right-hand side of (5) and these fields will correctly maintain energy conservation (Arnowitt and Nath 1977, private communication). Since the divergences identically vanish in flat space, no contribution will be made to the Maxwell field equations by these fields. It remains to be seen whether these fields will contribute to the total energy–momentum tensor and to the Lorentz group generators.

3. Solution of gauge supersymmetry equations

We want a solution of equations (4)–(7). Since we are particularly interested in whether or not the $K_{\mu\nu}$ and h fields contribute to the total energy–momentum tensor in (5), we will only require the $q_{\mu\nu}$ and h fields to zeroth order in G or equivalently to zeroth order in $1/m_G^2$. It would be inconsistent to work to higher order in calculating the contributions to $T_{\mu\nu(\text{total})}$ since fields of order G^2 have already been neglected in (5) as we mentioned above. This means that flat-space equations can be used to calculate $q_{\mu\nu}$, h, and $F_{\mu\nu}$.

We can write down the solution to (4) in flat space as

$$-F_{01}=F_{10}=e/r^2; \qquad \text{all other } F_{\mu\nu}=0. \tag{8}$$

We assume a $(-1, +1, +1, +1)$ signature for the metric. Using (8), equation (7) now becomes $\Box^2 h=e^2/r^4$. If we assume time independence we have the flat-space solution

$$h=(e^2/2r^2)-(A/r). \tag{9}$$

We have required h to go to zero as $r\to\infty$ so that terms which are only functions of the angles and a $\ln r$ term have been omitted. The A term in (9) is the solution of the homogeneous equation with A an arbitrary constant. A also contains a contribution proportional to $\lim_{r'\to 0}(1/r')=\infty$ due to our assumed (unphysical) point charge. We can avoid these difficulties by going to a more smeared out charge as we do below in our discussion of the Lorentz group generators.

Now that we have h, we can plug (8) and (9) into (6) and solve for $q_{\mu\nu}$. If we assume $q_{\mu\nu}$ is diagonal and we have flat space then (6) can be written as

$$-q''_{00}-q'_{00}\frac{2}{r}-m_G^2q_{00}=S_{00} \tag{10}$$

$$q''_{11}-\frac{2}{r}q'_{11}-q''^{\lambda}_{\lambda}-m_G^2q_{11}=S_{11} \tag{11}$$

$$-q''^{2}_{2}-\frac{1}{r}q'^{\lambda}_{\lambda}-m_G^2q_2{}^2=S_2{}^2 \tag{12}$$

$$q_2{}^2=q_3{}^3, \tag{13}$$

where $S_{\mu\nu}$ denotes the source terms on the right-hand side of (6). To lowest non-zero order in $1/m_G^2$ we have simply

$$q_{\mu\nu}=-S_{\mu\nu}/m_G^2. \tag{14}$$

We could write down a more exact solution to (10)–(13) but since G terms have already been thrown away because we are in flat space, higher-order $1/m_G^4$ terms cannot be consistently written down in the solution (14). The solution (14) for $q_{\mu\nu}$ implies that $K_{\mu\nu}(q)=0$ to lowest order in $1/m_G^2$ and we have that the contribution of $K_{\mu\nu}-\frac{1}{2}g_{\mu\nu}K^{\lambda}{}_{\lambda}$ to $T_{\mu\nu}$ in (5) to the required order is *zero*. Thus the $q_{\mu\nu}$ field causes no trouble and we shall forget it henceforth.

Let us return to the h field and calculate its contribution to the energy–momentum tensor in (5), namely

$$T_{\mu\nu(h)}=-(h_{;\mu\nu}-g_{\mu\nu}\Box^2h). \tag{15}$$

Using (9) gives

$$T_{\mu\nu(h)}=\frac{e^2}{2r^4}\begin{pmatrix}-2 & & & \\ & -4+4rA/e^2 & & \\ & & +4r^2-2r^3A/e^2 & \\ & & & +4r^2\sin^2\theta-2r^3A\sin^2\theta/e^2\end{pmatrix}. \tag{16}$$

We note that the A term in (9) does not contribute to T_{00} but does contribute to the other diagonal elements of $T_{\mu\nu}$. The energy–momentum term of the electromagnetic (EM) field for comparison is

$$T_{\mu\nu(\mathrm{EM})}=\frac{e^2}{2r^4}\begin{pmatrix}1 & & & \\ & -1 & & \\ & & r^2 & \\ & & & r^2\sin^2\theta\end{pmatrix}. \tag{17}$$

The total energy–momentum tensor which contributes to Einstein's equations is just (16) plus (17). In particular

$$T_{00(\mathrm{total})}=\frac{e^2}{2r^4}(-2+1)<0, \tag{18}$$

where the −2 is the h field contribution and the +1 is the EM field contribution. The h field contribution is clearly as large as the EM contribution and is negative. It certainly *cannot* be neglected in the symmetry broken field equations. We note that the h field equation (7) differs significantly from the $q_{\mu\nu}$ field equation (6) since (7) does not contain m_G^2 and (6) does. This is why $q_{\mu\nu}$ makes no contribution to $T_{\mu\nu(\mathrm{total})}$ but h does.

4. Discussion and conclusions

Equation (18) says that the h field makes a contribution to the energy–momentum tensor as large as and of the same form as the electromagnetic field contribution itself. Since (3) was used to manipulate (5) into a form where $T_{\mu\nu(\mathrm{EM})}$ appears as the source of Einstein's equations, it is clear that electromagnetism and gravitation have *not* been unified in a satisfactory way. In addition since $T_{00(\mathrm{total})}<0$ we have unphysical negative energy densities and the possibility of extracting an infinite amount of energy from a given volume. The Newtonian potential, of course, also has a reversed sign corresponding to a negative effective mass.

It is interesting to look at the contribution of the h field to the Lorentz group generators. In particular in flat space we have

$$p^0=\int \mathrm{d}^3x T^{00}_{(h)}=-\int \mathrm{d}^3x\, h^{;i}{}_{;i} \tag{19}$$

where the i is summed only over 1, 2, 3. Using Gauss's theorem to convert (19) into a surface integral then suggests that if h drops off fast enough for large r, there will be no contribution to p^0. Unfortunately for the point charge considered above, $h^{;i}{}_{;i}=e^2/r^4$. Integrating this over all space clearly gives a non-zero contribution in (19) *exactly* as the electromagnetic field does (the radial and charge dependences are identical). More generally, we can avoid the singular behaviour of our assumed point charge at the origin by writing the asymptotic solution of (7) as

$$h\simeq(8\pi r)^{-1}\int \mathrm{d}^3x'\, F^{\lambda\beta}(x')F_{\lambda\beta}(x'). \tag{20}$$

Then h clearly does not drop off fast enough at large r to avoid a contribution to (19).

The Arnowitt and Nath gauge supersymmetry formalism is almost compellingly elegant. The above serious difficulties suggest that something is wrong, not with the basic formalism, but with the detailed way in which the spontaneous symmetry breaking occurs. This is indeed found to be the case. Contrary to their published work, the most recent (unpublished) work of Arnowitt and Nath (1977) shows that the h field also grows a superheavy mass from spontaneous symmetry breaking exactly like $p_{\mu\nu}$. Thus (7) acquires a $m_G^2 h$ term and to lowest order h is proportional to $(1/m_G^2)F^{\lambda\beta}F_{\lambda\beta}$. Thus h becomes a higher-order field curing the difficulties cited above. The interesting point in the present paper, then, is that we have shown that this mass growth of h is *absolutely necessary* if we are to have an acceptable unification of gravitation and electromagnetism. The fact that this necessary mass growth follows from spontaneous symmetry breaking in this very close knit formalism is particularly satisfying.

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